Making an Irreversible Process Reversible

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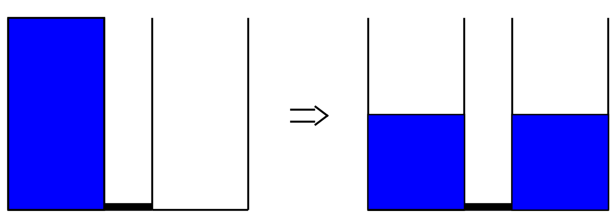
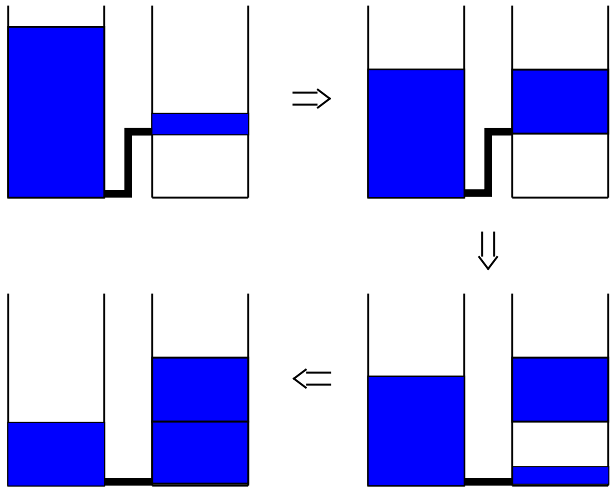
The concepts of reversible and irreversible processes are very important in thermodynamics but they are usually explained in rather abstract terms that take some getting used to. We will give what we think is a more intuitive example of such processes in the form of the simple problem of transferring liquid from one container to another.

We have two identical cylindrical liquid tanks sitting at the same level next to each other. One of them is full and the other is empty. The problem is to transfer as much liquid as possible from the full tank to the empty tank without moving either tank and keeping them both on the same level, and without any input of energy in the form of a pump.

If we just connect the bottom of the two tanks with a hose then liquid will flow from the full to the empty tank, until they both have the same liquid level. In this way, half of the liquid is transferred from the full to the empty tank, and this is clearly an irreversible process. You can't get all the liquid back into the full tank without the input of additional energy.



Is there any way to transfer more than half the liquid? The answer is yes, if we first partition the empty tank into two parts, one on top of the other, as shown in the following figure. The top part is filled first by connecting its bottom to the bottom of the full tank. The bottom part is then filled in the same way. If the height of the partition is correctly placed we can get more than half of the liquid into the empty tank.



So where exactly should we put this partition? To make things simple let's set the level of the full tank equal to 1. We will call the level of the partition y, and the final height of the liquid above the partition x. The following equation must then be satisfied.

$$1 - x = y + x$$

If we impose the condition that the bottom part must be filled completely then the following equation must also be satisfied

$$1 - x - y = y$$

Solving these equations for x and y we get $x = y = 1/3$. This means that if we place the partition at a height of 1/3 we can fill the top part to a height of 1/3 and fully fill the bottom part for a total height of 2/3 which is more than the 1/2 we get without partitioning.

It is fairly easy to show that this value for y is optimal. No other value of y will let you transfer more liquid. The way to transfer more liquid is to use more partitions. You can partition the second tank into 3 parts by placing the partitions at $y_1 = 1/2$ and $y_2 = 1/4$ which will allow you to transfer 3/4 of the liquid. If you partition into 4 parts by placing partitions at $(y_1, y_2, y_3) = (3/5, 2/5, 1/5)$ you can transfer 4/5 or 80% of the liquid.

In general partitioning the empty tank into N parts will allow $N/(N+1)$ of the liquid to be transferred. By making N large enough you can transfer as much of the liquid as you want and in the limit as $N \to \infty$ all of the liquid is transferred and the process becomes completely reversible. You can turn around and transfer all the liquid back to the original tank.

The question of reversibility is all about change in entropy and the loss of useful energy so let's look at the energies involved in the above processes. The gravitational potential energy stored in a cylindrical column of liquid is proportional to the square of the height of the column. For simplicity we'll assume the proportionality constant is equal to 1.

The full tank then starts out with energy $E = 1$. In the case of no partitioning the final height of the liquid in both tanks is $1/2$ so the final energy is $1/4 + 1/4 = 1/2$. Half the initial energy has been lost but where does it go? It must eventually be dissipated as heat into the surroundings causing a net increase in entropy. In the case of a partition into two parts, the two tanks have final liquid heights of $1/3$ and $2/3$ for a final energy of $1/9 + 4/9 = 5/9$. In this case only $4/9$ of the energy has disappeared and there is correspondingly a smaller increase in entropy.

In general for a partition into N parts the height of the liquid in the two columns will be $1/(N+1)$ and $N/(N+1)$ so that the energy is

$$E = \frac{N^2 + 1}{(N+1)^2}$$

In the limit $N \to \infty$ the energy is equal to the initial energy, $E = 1$. No energy is lost, no entropy is created and the process is completely reversible.

Another example of an irreversible process is the transfer of heat from a quantity of water at temperature T_1 to another quantity of water at temperature $T_2 < T_1$. If you split up the T_2 water and put each piece in contact with the T_1 water sequentially you can lower the entropy production and transfer more of the heat i.e. the process becomes less irreversible. This is the basic principle used in heat exchangers. For an interesting take on this idea see the paper by Mischenko and Pshenichka listed in the references below.

References

Eugene G. Mishchenko, Paul F. Pshenichka, “Reversible temperature exchange upon thermal contact”, American Journal of Physics, Vol 85, No 1, January 2017, p23-29.